OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4757

Further Applications of Advanced Mathematics (FP3)

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 13 June 2011
Morning

Duration: 1 hour 30 minutes

4757

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Option 1: Vectors

1 The points A (2, −1, 3), B (−2, −7, 7) and C (7, 5, 1) are three vertices of a tetrahedron ABCD.

The plane ABD has equation $x + 4y + 7z = 19$.

The plane ACD has equation $2x - y + 2z = 11$.

(i) Find the shortest distance from B to the plane ACD. **[3]**

(ii) Find an equation for the line AD. **[3]**

(iii) Find the shortest distance from C to the line AD. **[6]**

(iv) Find the shortest distance between the lines AD and BC. **[6]**

(v) Given that the tetrahedron ABCD has volume 20, find the coordinates of the two possible positions for the vertex D. **[6]**

Option 2: Multi-variable calculus

2 A surface S has equation $z = 8y^3 - 6x^2y - 15x^2 + 36x$.

(i) Sketch the section of S given by $y = -3$, and sketch the section of S given by $x = -6$. Your sketches should include the coordinates of any stationary points but need not include the coordinates of the points where the sections cross the axes. **[7]**

(ii) From your sketches in part **(i)**, deduce that (−6, −3, −324) is a stationary point on S, and state the nature of this stationary point. **[2]**

(iii) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, and hence find the coordinates of the other three stationary points on S. **[8]**

(iv) Show that there are exactly two values of k for which the plane with equation

$$120x - z = k$$

is a tangent plane to S, and find these values of k. **[7]**

Option 3: Differential geometry

3 **(a)** **(i)** Given that $y = \mathrm{e}^{\frac{1}{2}x} + \mathrm{e}^{-\frac{1}{2}x}$, show that $1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \left(\frac{1}{2}\mathrm{e}^{\frac{1}{2}x} + \frac{1}{2}\mathrm{e}^{-\frac{1}{2}x}\right)^2$. **[3]**

The arc of the curve $y = \mathrm{e}^{\frac{1}{2}x} + \mathrm{e}^{-\frac{1}{2}x}$ for $0 \leqslant x \leqslant \ln a$ (where $a > 1$) is denoted by C.

(ii) Show that the length of C is $\frac{a-1}{\sqrt{a}}$. **[3]**

(iii) Find the area of the surface formed when C is rotated through 2π radians about the x-axis. **[5]**

(b) An ellipse has parametric equations $x = 2\cos\theta$, $y = \sin\theta$ for $0 \leqslant \theta < 2\pi$.

(i) Show that the normal to the ellipse at the point with parameter θ has equation

$$y = 2x\tan\theta - 3\sin\theta.$$ **[3]**

(ii) Find parametric equations for the evolute of the ellipse, and show that the evolute has cartesian equation

$$(2x)^{\frac{2}{3}} + y^{\frac{2}{3}} = 3^{\frac{2}{3}}.$$ **[6]**

(iii) Using the evolute found in part **(ii)**, or otherwise, find the radius of curvature of the ellipse

(A) at the point (2, 0),

(B) at the point (0, 1). **[4]**

Option 4: Groups

4 **(i)** Show that the set $G = \{1, 3, 4, 5, 9\}$, under the binary operation of multiplication modulo 11, is a group. You may assume associativity. **[6]**

(ii) Explain why any two groups of order 5 must be isomorphic to each other. **[3]**

The set $H = \left\{1, e^{\frac{2}{5}\pi j}, e^{\frac{4}{5}\pi j}, e^{\frac{6}{5}\pi j}, e^{\frac{8}{5}\pi j}\right\}$ is a group under the binary operation of multiplication of complex numbers.

(iii) Specify an isomorphism between the groups G and H. **[3]**

The set K consists of the 25 ordered pairs (x, y), where x and y are elements of G. The set K is a group under the binary operation defined by

$$(x_1, y_1)(x_2, y_2) = (x_1x_2, y_1y_2)$$

where the multiplications are carried out modulo 11; for example, $(3, 5)(4, 4) = (1, 9)$.

(iv) Write down the identity element of K, and find the inverse of the element $(9, 3)$. **[2]**

(v) Explain why $(x, y)^5 = (1, 1)$ for every element (x, y) in K. **[3]**

(vi) Deduce that all the elements of K, except for one, have order 5. State which is the exceptional element. **[3]**

(vii) A subgroup of K has order 5 and contains the element $(9, 3)$. List the elements of this subgroup. **[2]**

(viii) Determine how many subgroups of K there are with order 5. **[2]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

Alpha and Delta are two companies which compete for the ownership of insurance bonds. Boyles and Cayleys are companies which trade in these bonds. When a new bond becomes available, it is first acquired by either Boyles or Cayleys. After a certain amount of trading it is eventually owned by either Alpha or Delta. Change of ownership always takes place overnight, so that on any particular day the bond is owned by one of the four companies. The trading process is modelled as a Markov chain with four states, as follows.

On the first day, the bond is owned by Boyles or Cayleys, with probabilities 0.4, 0.6 respectively.

If the bond is owned by Boyles, then on the next day it could be owned by Alpha, Boyles or Cayleys, with probabilities 0.07, 0.8, 0.13 respectively.

If the bond is owned by Cayleys, then on the next day it could be owned by Boyles, Cayleys or Delta, with probabilities 0.15, 0.75, 0.1 respectively.

If the bond is owned by Alpha or Delta, then no further trading takes place, so on the next day it is owned by the same company.

(i) Write down the transition matrix **P**. **[2]**

(ii) Explain what is meant by an absorbing state of a Markov chain. Identify any absorbing states in this situation. **[2]**

(iii) Find the probability that the bond is owned by Boyles on the 10th day. **[3]**

(iv) Find the probability that on the 14th day the bond is owned by the same company as on the 10th day. **[3]**

(v) Find the day on which the probability that the bond is owned by Alpha or Delta exceeds 0.9 for the first time. **[4]**

(vi) Find the limit of $\mathbf{P}^n$ as n tends to infinity. **[2]**

(vii) Find the probability that the bond is eventually owned by Alpha. **[3]**

The probabilities that Boyles and Cayleys own the bond on the first day are changed (but all the transition probabilities remain the same as before). The bond is now equally likely to be owned by Alpha or Delta at the end of the trading process.

(viii) Find the new probabilities for the ownership of the bond on the first day. **[5]**

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GCE

Mathematics (MEI)

Advanced GCE

Unit **4757:** Further Applications of Advanced Mathematics

Mark Scheme for June 2011

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GENERAL MARKING INSTRUCTIONS

1 Please mark in RED.

2 Errors in the scripts should be clearly indicated (e.g. by ringing them). Please tick correct work. Some indication <u>must</u> be given on every page that the work has been assessed.

3 Marks for sections of questions should be written in the right-hand margin; detailed marks (e.g. M1A0M1A1) need only be shown when the breakdown of marks is not obvious.

4 The total mark for each question should be written in the right-hand margin at the end of the answer, and ringed.

5 Work crossed out by the candidate, and not replaced, should be marked wherever possible.

6 M: method marks

A: accuracy marks which are dependent on the relevant previous method mark(s) having been awarded (e.g. marks M0A1 cannot be awarded; where appropriate, the correct answer obtained with no method shown is taken to imply the method marks).

B: accuracy marks which are not dependent on method marks.

7 Marks are indivisible; if the scheme says M2, give either 2 or 0, except where indicated in the scheme.

8 Where there are two or more method marks for a section of work, the method marks can be awarded independently, except where indicated in the scheme.

9 cao: correct answer only (i.e. no follow through).

All A and B marks are cao unless stated otherwise.

ft: award mark for correct working following through from a previous error; use ✓^ to indicate correct ft work, and ✗^ to indicate that a further error has been made. Ft marks are intended to ensure that a small arithmetic error is not unduly penalised; when the previous error is one of principle, particularly if the nature of the work is changed, or made considerably easier, the ft might not be given.

10 'Correctly obtained' means that all the working leading to that result must be correct.

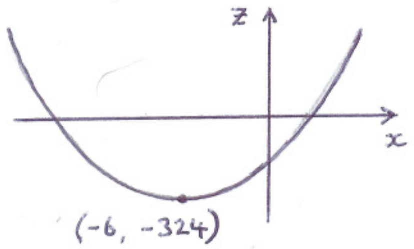
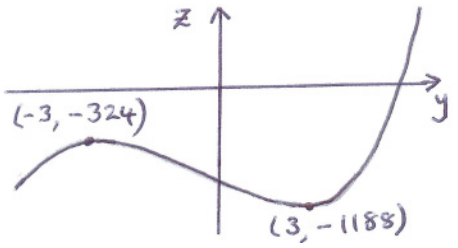
11 Where the candidate is asked to show a given result, we expect the explanation to be reasonably clear.

12 If the candidate misreads a question, but in such a way that the nature and difficulty of the work is not changed, transfer all marks (including cao) to the new equivalent figures. Deduct the first A or B mark so earned (M marks are not lost for misread).

1 (i)	Distance is $\dfrac{2(-2)-(-7)+2(7)-11}{\sqrt{2^2+1^2+2^2}}$ $=2$	M1 A1 A1 **3**	Formula, or other complete method Numerical expression for distance
(ii)	$\begin{pmatrix}1\\4\\7\end{pmatrix}\times\begin{pmatrix}2\\-1\\2\end{pmatrix}=\begin{pmatrix}15\\12\\-9\end{pmatrix}$ Equation of AD is $\mathbf{r}=\begin{pmatrix}2\\-1\\3\end{pmatrix}+\lambda\begin{pmatrix}5\\4\\-3\end{pmatrix}$	M1 A1 A1 ft **3**	Vector product of normals, or finding a point on AD, e.g. $(0,-2.6,4.2)$, $(3.25,0,2.25)$, $(7,3,0)$ Correct direction Accept any form
(iii)	$\overrightarrow{\mathrm{CA}}\times\mathbf{d}=\begin{pmatrix}-5\\-6\\2\end{pmatrix}\times\begin{pmatrix}5\\4\\-3\end{pmatrix}=\begin{pmatrix}10\\-5\\10\end{pmatrix}$ Distance is $\dfrac{\lvert\overrightarrow{\mathrm{CA}}\times\mathbf{d}\rvert}{\lvert\mathbf{d}\rvert}=\dfrac{\sqrt{10^2+5^2+10^2}}{\sqrt{5^2+4^2+3^2}}=\dfrac{\sqrt{225}}{\sqrt{50}}$ $=\sqrt{4.5}=\dfrac{3}{\sqrt{2}}=\dfrac{3\sqrt{2}}{2}\approx 2.12$	M1 A2 ft M1 M1 A1 **6**	Appropriate vector product Give A1 if just one error Formula for distance Finding magnitude *Both dependent on first M1*
(iv)	$\mathbf{d}\times\overrightarrow{\mathrm{BC}}=\begin{pmatrix}5\\4\\-3\end{pmatrix}\times\begin{pmatrix}9\\12\\-6\end{pmatrix}=\begin{pmatrix}12\\3\\24\end{pmatrix}=3\begin{pmatrix}4\\1\\8\end{pmatrix}$ Distance is $\dfrac{\begin{pmatrix}-4\\-6\\4\end{pmatrix}.\begin{pmatrix}4\\1\\8\end{pmatrix}}{\sqrt{4^2+1^2+8^2}}=\dfrac{10}{9}$	M1 A1 ft M1 A1 ft M1 A1 **6**	Vector product of directions Appropriate scalar product Evaluation of scalar product For denominator
(v)	$V=\frac{1}{6}(\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AC}}).\overrightarrow{\mathrm{AD}}=\frac{1}{6}\left[\begin{pmatrix}-4\\-6\\4\end{pmatrix}\times\begin{pmatrix}5\\6\\-2\end{pmatrix}\right].\lambda\begin{pmatrix}5\\4\\-3\end{pmatrix}$ $=\frac{1}{6}\lambda\begin{pmatrix}-12\\12\\6\end{pmatrix}.\begin{pmatrix}5\\4\\-3\end{pmatrix}=-5\lambda$ $V=\pm 20 \Rightarrow \lambda=\pm 4$ D is $(22,15,-9)$ or $(-18,-17,15)$	M1 M1 A1 ft M1 A1A1 **6**	Appropriate scalar triple product Evaluation of scalar triple product or $-2a+2b+c+3$ (simplified) for $\mathrm{D}(a,b,c)$ Obtain a value of λ, or one of a, b, c

Alternative methods for Question 1

1 (ii)	Eliminating x $3y+4z=9$ $x=7-\frac{5}{3}t,\ \ y=3-\frac{4}{3}t,\ \ z=t$	M1 A1A1 **3**	Eliminating one of x, y, z or $3x+5z=21$ or $4x-5y=13$
1 (iii)	$\left[\begin{pmatrix}2\\-1\\3\end{pmatrix}+\lambda\begin{pmatrix}5\\4\\-3\end{pmatrix}-\begin{pmatrix}7\\5\\1\end{pmatrix}\right].\begin{pmatrix}5\\4\\-3\end{pmatrix}=0$ $25\lambda-25+16\lambda-24+9\lambda-6=0$ $\lambda=1.1$, F is $(7.5, 3.4, -0.3)$ $\text{CF}=\sqrt{(0.5)^2+(-1.6)^2+(-1.3)^2}$ $=\sqrt{4.5}$	M1 A1 ft A1 ft M1 M1 A1 **6**	Appropriate scalar product Obtaining a value for λ Finding magnitude
1 (iv)	$\left[\begin{pmatrix}-2\\-7\\7\end{pmatrix}+\mu\begin{pmatrix}9\\12\\-6\end{pmatrix}-\begin{pmatrix}2\\-1\\3\end{pmatrix}-\lambda\begin{pmatrix}5\\4\\-3\end{pmatrix}\right].\begin{pmatrix}5\\4\\-3\end{pmatrix}=0$ and $\begin{pmatrix}-5\lambda+9\mu-4\\-4\lambda+12\mu-6\\3\lambda-6\mu+4\end{pmatrix}.\begin{pmatrix}9\\12\\-6\end{pmatrix}=0$ $\lambda=\frac{4}{81},\quad \mu=\frac{128}{243}$ Distance is $\sqrt{\left(\frac{40}{81}\right)^2+\left(\frac{10}{81}\right)^2+\left(\frac{80}{81}\right)^2}$ $=\frac{10}{9}$	M1 A1 ft A1 ft M1 M1 A1 **6**	Two appropriate scalar products Obtaining values for λ and μ Obtaining distance

2 (i)	When $y=-3,\quad z=3x^2+36x-216$  	B1 B1 B1		 Correct shape (parabola) and position For $(-6,-324)$
	When $x=-6,\quad z=8y^3-216y-756$  	B1 B1 B1 B1	7	 Correct shape and position For $(-3,-324)$ For $(3,-1188)$ *If B0B0 then give B1 for* $x=\pm 3$
(ii)	$(-6,-3,-324)$ is a SP on both sections; hence it is a SP on S Saddle point	B1 B1	2	
(iii)	$\frac{\partial z}{\partial x}=-12xy-30x+36,\quad \frac{\partial z}{\partial y}=24y^2-6x^2$ At a SP, $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}=0$ $24y^2-6x^2=0 \Rightarrow y=\pm\frac{1}{2}x$ $y=\frac{1}{2}x \Rightarrow -6x^2-30x+36=0$ $\Rightarrow x=-6,\ 1;$ SPs are $(-6,-3,-324)$ $(1,\ 0.5,\ 19)$ $y=-\frac{1}{2}x \Rightarrow 6x^2-30x+36=0$ $\Rightarrow x=2,\ 3;$ SPs are $(2,-1,\ 28)$ $(3,-1.5,\ 27)$	B1B1 M1 M1 A1 M1 A1 A1	8	
(iv)	$\frac{\partial z}{\partial x}=120$ and $\frac{\partial z}{\partial y}=0$ $y=\frac{1}{2}x \Rightarrow -6x^2-30x-84=0;\ D=30^2-4\times 6\times 84$ $D\ (=-1116)<0$; so this has no roots $y=-\frac{1}{2}x \Rightarrow 6x^2-30x-84=0 \Rightarrow x=7,-2$ When $x=7,\ y=-3.5,\ z=203;$ so $k=637$ When $x=-2,\ y=1,\ z=-148;$ so $k=-92$	M1 M1 A1 M1 M1 A1 A1	7	(Allow M1 for $\frac{\partial z}{\partial x}=-120$) Obtaining at least one value of x Obtaining a value of k

3(a)(i)	$\dfrac{dy}{dx}=\frac{1}{2}e^{\frac{1}{2}x}-\frac{1}{2}e^{-\frac{1}{2}x}$	B1	
	$\left(\dfrac{dy}{dx}\right)^2=\frac{1}{4}e^{x}-\frac{1}{2}+\frac{1}{4}e^{-x}$	M1	
	$1+\left(\dfrac{dy}{dx}\right)^2=\frac{1}{4}e^{x}+\frac{1}{2}+\frac{1}{4}e^{-x}=(\frac{1}{2}e^{\frac{1}{2}x}+\frac{1}{2}e^{-\frac{1}{2}x})^2$	A1 (ag) **3**	Correct completion
(ii)	Length is $\displaystyle\int_0^{\ln a}(\frac{1}{2}e^{\frac{1}{2}x}+\frac{1}{2}e^{-\frac{1}{2}x})\,dx$	M1	For $\int(\frac{1}{2}e^{\frac{1}{2}x}+\frac{1}{2}e^{-\frac{1}{2}x})\,dx$
	$=\left[e^{\frac{1}{2}x}-e^{-\frac{1}{2}x}\right]_0^{\ln a}$	A1	For $e^{\frac{1}{2}x}-e^{-\frac{1}{2}x}$
	$=\left(\sqrt{a}-\dfrac{1}{\sqrt{a}}\right)-(1-1)=\dfrac{a-1}{\sqrt{a}}$	A1 (ag) **3**	Correctly shown
(iii)	Curved surface area is $\int 2\pi y\,ds$	M1	For $\int y\,ds$
	$=\displaystyle\int_0^{\ln a}2\pi(e^{\frac{1}{2}x}+e^{-\frac{1}{2}x})(\frac{1}{2}e^{\frac{1}{2}x}+\frac{1}{2}e^{-\frac{1}{2}x})\,dx$	A1	Correct integral form *including limits*
	$=\pi\displaystyle\int_0^{\ln a}(e^{x}+2+e^{-x})\,dx$	M1	Obtaining integrable expression
	$=\pi\left[e^{x}+2x-e^{-x}\right]_0^{\ln a}$	A1	For $e^{x}+2x-e^{-x}$
	$=\pi\left(a+2\ln a-\dfrac{1}{a}\right)$	A1 **5**	
(b)(i)	$\dfrac{dy}{dx}=\dfrac{\cos\theta}{-2\sin\theta}$	B1	
	Gradient of normal is $\dfrac{2\sin\theta}{\cos\theta}$ $(=2\tan\theta)$	M1	
	Normal is $y-\sin\theta=2\tan\theta(x-2\cos\theta)$ $y=2x\tan\theta-3\sin\theta$	A1 (ag) **3**	Correctly shown
(ii)	Differentiating partially w.r.t. θ	M1	
	$0=2x\sec^2\theta-3\cos\theta$		
	$x=\frac{3}{2}\cos^3\theta$	A1	
	$y=3\cos^3\theta\tan\theta-3\sin\theta$	M1 A1	Obtaining an expression for y Any correct form
	$=3\sin\theta(\cos^2\theta-1)=-3\sin^3\theta$	M1	Using $1-\cos^2\theta=\sin^2\theta$
	$(2x)^{\frac{2}{3}}+y^{\frac{2}{3}}=(3\cos^3\theta)^{\frac{2}{3}}+(-3\sin^3\theta)^{\frac{2}{3}}$ $=3^{\frac{2}{3}}(\cos^2\theta+\sin^2\theta)=3^{\frac{2}{3}}$	A1 (ag) **6**	Correctly shown
(iii) (*A*)	$(2,0)$ has $\theta=0$ Centre of curvature is $(\frac{3}{2},\ 0)$	M1	Using param eqn with $\theta=0$ *(or other method for ρ or cc)*
	$\rho=\frac{1}{2}$	A1	
(*B*)	$(0,1)$ has $\theta=\frac{1}{2}\pi$ Centre of curvature is $(0,\ -3)$	M1	Using param eqn with $\theta=\frac{1}{2}\pi$ *(or other method for ρ or cc)*
	$\rho=4$	A1 **4**	

<table>
<tr><td>4 (i)</td><td>
<table>
<tr><td></td><td>1</td><td>3</td><td>4</td><td>5</td><td>9</td></tr>
<tr><td>1</td><td>1</td><td>3</td><td>4</td><td>5</td><td>9</td></tr>
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<tr><td>5</td><td>5</td><td>4</td><td>9</td><td>3</td><td>1</td></tr>
<tr><td>9</td><td>9</td><td>5</td><td>3</td><td>1</td><td>4</td></tr>
</table>
Composition table shows closure
Identity is 1
<table>
<tr><td>Element</td><td>1</td><td>3</td><td>4</td><td>5</td><td>9</td></tr>
<tr><td>Inverse</td><td>1</td><td>4</td><td>3</td><td>9</td><td>5</td></tr>
</table>
So every element has an inverse</td><td>B2

B1
B1

B2
6</td><td>Give B1 if not more than 4 errors

Dependent on B2 for table

Give B1 for 3 correct</td></tr>
<tr><td>(ii)</td><td>Since 5 is prime,
a group of order 5 must be cyclic
Two cyclic groups of the same order must be isomorphic</td><td>B1
B1

B1
3</td><td></td></tr>
<tr><td>(iii)</td><td>
<table>
<tr><td>H</td><td>1</td><td>$e^{\frac{2}{5}\pi j}$</td><td>$e^{\frac{4}{5}\pi j}$</td><td>$e^{\frac{6}{5}\pi j}$</td><td>$e^{\frac{8}{5}\pi j}$</td></tr>
<tr><td>G</td><td>1</td><td>3</td><td>9</td><td>5</td><td>4</td></tr>
<tr><td>or</td><td>1</td><td>4</td><td>5</td><td>9</td><td>3</td></tr>
<tr><td>or</td><td>1</td><td>5</td><td>3</td><td>4</td><td>9</td></tr>
<tr><td>or</td><td>1</td><td>9</td><td>4</td><td>3</td><td>5</td></tr>
</table></td><td>B1

B2
3</td><td>For $1 \leftrightarrow 1$

For non-identity elements</td></tr>
<tr><td>(iv)</td><td>Identity is $(1,\ 1)$
Inverse of $(9,\ 3)$ is $(5,\ 4)$</td><td>B1
B1
2</td><td></td></tr>
<tr><td>(v)</td><td>$(x,\ y)^5 = (x^5,\ y^5)$
Since G has order 5, $x^5 = 1$ and $y^5 = 1$
Hence $(x,\ y)^5 = (1,\ 1)$</td><td>M1
M1
A1 (ag)
3</td><td></td></tr>
<tr><td>(vi)</td><td>Order of $(x,\ y)$ is a factor of 5 (so must be 1 or 5)
Only identity $(1,\ 1)$ can have order 1
Hence all other elements have order 5</td><td>M1
B1
A1 (ag)
3</td><td></td></tr>
<tr><td>(vii)</td><td>$\{(1,\ 1),\ (9,\ 3),\ (4,\ 9),\ (3,\ 5),\ (5,\ 4)\}$</td><td>B2
2</td><td>Give B1 ft for 5 elements including $(1,\ 1),\ (9,\ 3),\ (5,\ 4)$</td></tr>
<tr><td>(viii)</td><td>An element of order 5 generates a subgroup, and so can be in only one subgroup of order 5

Number is $24 \div 4 = 6$</td><td>M1

A1
2</td><td>Or for $24 \div 4$
Or listing at least 2 other subgroups
Give B1 for unsupported answer 6</td></tr>
</table>

Pre-multiplication by transition matrix

5 (i)	$\mathbf{P} = \begin{pmatrix} 1 & 0.07 & 0 & 0 \\ 0 & 0.8 & 0.15 & 0 \\ 0 & 0.13 & 0.75 & 0 \\ 0 & 0 & 0.1 & 1 \end{pmatrix}$	B2 **2**	*Allow tolerance of* ± 0.0001 *in probabilities throughout this question* Give B1 for two columns correct
(ii)	If system enters an absorbing state, it remains in that state A and D are absorbing states	B1 B1 **2**	
(iii)	$\mathbf{P}^9 \begin{pmatrix} 0 \\ 0.4 \\ 0.6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.2236 \\ 0.2505 \\ 0.1998 \\ 0.3261 \end{pmatrix}$ Prob(owned by B) = 0.2505	M1M1 A1 **3**	For $\mathbf{P}^9$ (or $\mathbf{P}^{10}$) and $\begin{pmatrix} 0 \\ 0.4 \\ 0.6 \\ 0 \end{pmatrix}$
(iv)	$\mathbf{P}^4 = \begin{pmatrix} 1 & \dots & \dots & \dots \\ \dots & 0.4818 & \dots & \dots \\ \dots & \dots & 0.3856 & \dots \\ \dots & \dots & \dots & 1 \end{pmatrix}$ $0.2236 + 0.2505 \times 0.4818 + 0.1998 \times 0.3856 + 0.3261$ $= 0.7474$	M1 M1 A1 **3**	Using diagonal elements from $\mathbf{P}^4$ Using probabilities for 10th day
(v)	$\begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix} \mathbf{P}^n \begin{pmatrix} 0 \\ 0.4 \\ 0.6 \\ 0 \end{pmatrix}$ $= (\ 0.8971\ )$ when $n = 26$ $= (\ 0.9057\ )$ when $n = 27$ i.e. on the 28th day	M1 M1 A1 ft A1 **4**	Considering $\mathbf{P}^n$ for some $n > 20$ Evaluating Prob(A or D) for some values of n Identifying $n = 26$ or $n = 27$ (*Implies M1M1*)
(vi)	$\mathbf{P}^n \to \begin{pmatrix} 1 & \mathbf{0.5738} & \mathbf{0.3443} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \mathbf{0.4262} & \mathbf{0.6557} & 1 \end{pmatrix} = \mathbf{Q}$	B2 **2**	Give B1 for two bold elements correct (to 3 dp)
(vii)	$\mathbf{Q} \begin{pmatrix} 0 \\ 0.4 \\ 0.6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.4361 \\ 0 \\ 0 \\ 0.5639 \end{pmatrix}$ Prob(eventually owned by A) = 0.4361	M1M1 A1 **3**	Using $\mathbf{Q}$ and $\begin{pmatrix} 0 \\ 0.4 \\ 0.6 \\ 0 \end{pmatrix}$
(viii)	$\mathbf{Q} \begin{pmatrix} 0 \\ p \\ q \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0 \\ 0 \\ 0.5 \end{pmatrix}$ (where $q = 1 - p$) $0.5738p + 0.3443q = 0.5$ $p = 0.6786, \quad q = 0.3214$	M1M1 A1 ft M1 A1 **5**	For LHS and RHS Or $0.4262p + 0.6557q = 0.5$ Solving to obtain a value of p Allow 0.678 - 0.679, 0.321 – 0.322

Post-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0.07 & 0.8 & 0.13 & 0\\ 0 & 0.15 & 0.75 & 0.1\\ 0 & 0 & 0 & 1\end{pmatrix}$	B2 **2**	*Allow tolerance of* ± 0.0001 *in probabilities throughout this question* Give B1 for two rows correct
(ii)	If system enters an absorbing state, it remains in that state A and D are absorbing states	B1 B1 **2**	
(iii)	$\begin{pmatrix}0 & 0.4 & 0.6 & 0\end{pmatrix}\mathbf{P}^9$ $=\begin{pmatrix}0.2236 & 0.2505 & 0.1998 & 0.3261\end{pmatrix}$ Prob(owned by B) = 0.2505	M1M1 A1 **3**	$\begin{pmatrix}0 & 0.4 & 0.6 & 0\end{pmatrix}$ and $\mathbf{P}^9$ (or $\mathbf{P}^{10}$)
(iv)	$\mathbf{P}^4=\begin{pmatrix}1 & ... & ... & ...\\ ... & 0.4818 & ... & ...\\ ... & ... & 0.3856 & ...\\ ... & ... & ... & 1\end{pmatrix}$ $0.2236+0.2505\times 0.4818+0.1998\times 0.3856+0.3261$ $= 0.7474$	M1 M1 A1 **3**	Using diagonal elements from $\mathbf{P}^4$ Using probabilities for 10th day
(v)	$\begin{pmatrix}0 & 0.4 & 0.6 & 0\end{pmatrix}\mathbf{P}^n\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$ $=(\ 0.8971\ )$ when $n=26$ $=(\ 0.9057\ )$ when $n=27$ i.e. on the 28th day	M1 M1 A1 ft A1 **4**	Considering $\mathbf{P}^n$ for some $n>20$ Evaluating Prob(A or D) for some values of n Identifying $n=26$ or $n=27$ *(Implies M1M1)*
(vi)	$\mathbf{P}^n \rightarrow \begin{pmatrix}1 & 0 & 0 & 0\\ \mathbf{0.5738} & 0 & 0 & \mathbf{0.4262}\\ \mathbf{0.3443} & 0 & 0 & \mathbf{0.6557}\\ 0 & 0 & 0 & 1\end{pmatrix}=\mathbf{Q}$	B2 **2**	Give B1 for two bold elements correct (to 3 dp)
(vii)	$\begin{pmatrix}0 & 0.4 & 0.6 & 0\end{pmatrix}\mathbf{Q}=\begin{pmatrix}0.4361 & 0 & 0 & 0.5639\end{pmatrix}$ Prob(eventually owned by A) = 0.4361	M1M1 A1 **3**	Using $\begin{pmatrix}0 & 0.4 & 0.6 & 0\end{pmatrix}$ and $\mathbf{Q}$
(viii)	$\begin{pmatrix}0 & p & q & 0\end{pmatrix}\mathbf{Q}=\begin{pmatrix}0.5 & 0 & 0 & 0.5\end{pmatrix}$ (where $q=1-p$) $0.5738p+0.3443q=0.5$ $p=0.6786,\quad q=0.3214$	M1M1 A1 ft M1 A1 **5**	For LHS and RHS Or $0.4262p+0.6557q=0.5$ Solving to obtain a value of p Allow 0.678 - 0.679, 0.321 – 0.322

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4757: Further Applications of Advanced Mathematics (FP3)

General Comments

The work on this paper was generally of a high standard, and there were many excellent scripts, with about a third of the candidates scoring 60 marks or more (out of 72). Almost all candidates made substantial attempts at three questions, with just a very few answering an additional fourth question. The most popular question was question 1, followed by question 2 then question 5, question 4 and question 3. The average marks achieved on the five questions were similar, ranging from about 16 (out of 24) on questions 1, 3 and 4 to about 18 on question 5. A fair number of candidates were clearly rushed at the end and were unable to complete their final question; this was almost always caused by spending too long using over-complicated methods in question 1.

Comments on Individual Questions

1 *(Vectors)*
In part (i) the perpendicular distance from a point to a plane was usually found correctly. Part (ii) was very often not immediately recognised as the intersection of two planes, with lengthy, and often incorrect, methods being attempted instead. This work was frequently crossed out and a more appropriate method, such as finding the vector product of the two normals, was then adopted. However, several candidates were unable to obtain any answer for this part, and many gave a wrong equation; then using their incorrect answer they could obtain most of the marks in the remainder of the question. In parts (iii) and (iv) most candidates used efficient methods to find the shortest distances from a point to a line and between two skew lines, although in part (iii) a fairly common error was to use a scalar product instead of the vector product in the formula $\left| \overrightarrow{\mathbf{AC}} \times \overrightarrow{\mathbf{AD}} \right| / \left| \overrightarrow{\mathbf{AD}} \right|$. In part (v) almost all candidates knew that the volume of the tetrahedron was given by the scalar triple product $\frac{1}{6}(\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}}) . \overrightarrow{\mathbf{AD}}$ (although the $\frac{1}{6}$ was sometimes omitted). However, $\overrightarrow{\mathbf{AD}}$ was often put equal to the position vector of D. Very many candidates used the coordinates of D$(x,\ y,\ z)$ to obtain the correct equation $-2x + 2y + z + 3 = \pm 20$ but were unable to proceed beyond this.

2 *(Multi-variable calculus)*
The section sketches and stationary points in parts (i) and (ii) were generally well understood, although careless slips spoilt many answers. In part (iii) the partial derivatives were almost always found correctly and equated to zero. The equation $24y^2 - 6x^2 = 0$ was quite often simplified to $x = 2y$ only (omitting the case $x = -2y$), and arithmetic and sign errors were fairly common. Several candidates used the quadratic substitution and obtained a quartic equation, which was sometimes solved successfully. In part (iv) most candidates used the partial derivatives to obtain two correct equations, but only a few managed to score full marks in this part. Some did not state that the quadratic equation obtained from $x = 2y$ had no real roots, and those who omitted the case $x = -2y$ were unable to find any values. Some obtained values for x or y, but did not go on to find the values of k.

3 *(Differential geometry)*
Part (a) was answered well, with the arc length and surface area usually being found correctly, although there were many algebraic errors, for example when multiplying out. In part (b)(i) the equation of the normal was usually obtained correctly. Part (b)(ii) was quite often omitted, but when attempted the parametric equations for the evolute were generally found correctly and frequently used to obtain the cartesian equation. The final part (b)(iii) was very often not attempted at all. It was intended that candidates would use the parametric equations of the evolute to find the centres of curvature, but most preferred to use the standard formulae and many were successful.

4 *(Groups)*
Most candidates showed very good understanding in parts (i), (iv) and (vii). Only about half gave a correct isomorphism in part (iii) and the final part (viii) defeated most candidates. The explanations in parts (ii), (v), and especially (vi), were very often inadequate, and only a handful of candidates scored full marks on this question. In part (i) explicit reference to the group properties of closure, identity and inverses was expected, and most candidates did this nicely by exhibiting the complete composition table and listing the inverses. Those for example who showed that all the elements could be written as powers of 3 were expected to explain why this implies closure and the existence of inverses. In part (v) the step $(x, y)^5 = (x^5, y^5)$ was quite often omitted, and in part (vi) the essential point that $(x, y)^5 = (1, 1)$ implies the order of (x, y) is a factor of 5 was rarely stated clearly.

5 *(Markov chains)*
The techniques were generally well understood and calculators were used competently throughout this question; parts (i), (ii), (iii), (v), (vi) and (vii) were all answered very well. In part (iv) many candidates used the probabilities for days 10 and 14 instead of the probabilities for day 10 and the diagonal elements of $\mathbf{P}^4$. In part (viii) many candidates used the original transition matrix $\mathbf{P}$ instead of the limiting matrix found in part (vi); and a surprising number obtained correct equations for the new probabilities but could not solve them accurately.

GCE Mathematics (MEI)

			Max Mark	a	b	c	d	e	u
4751/01	(C1) MEI Introduction to Advanced Mathematics	Raw	72	55	49	43	37	32	0
		UMS	100	80	70	60	50	40	0
4752/01	(C2) MEI Concepts for Advanced Mathematics	Raw	72	53	46	39	33	27	0
		UMS	100	80	70	60	50	40	0
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	48	42	36	29	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	63	56	50	44	38	0
		UMS	100	80	70	60	50	40	0
4755/01	(FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	59	52	45	39	33	0
		UMS	100	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	55	48	42	36	30	0
		UMS	100	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	39	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01	(M1) MEI Mechanics 1	Raw	72	60	52	44	36	28	0
		UMS	100	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	64	57	51	45	39	0
		UMS	100	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	59	51	43	35	27	0
		UMS	100	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	54	47	40	33	26	0
		UMS	100	80	70	60	50	40	0
4766/01	(S1) MEI Statistics 1	Raw	72	53	45	38	31	24	0
		UMS	100	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	60	53	46	39	33	0
		UMS	100	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771/01	(D1) MEI Decision Mathematics 1	Raw	72	51	45	39	33	27	0
		UMS	100	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	58	53	48	43	39	0
		UMS	100	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776/01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	62	55	49	43	36	0
4776/02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776	(NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
		UMS	100	80	70	60	50	40	0